FORCES

15-16 year-olds

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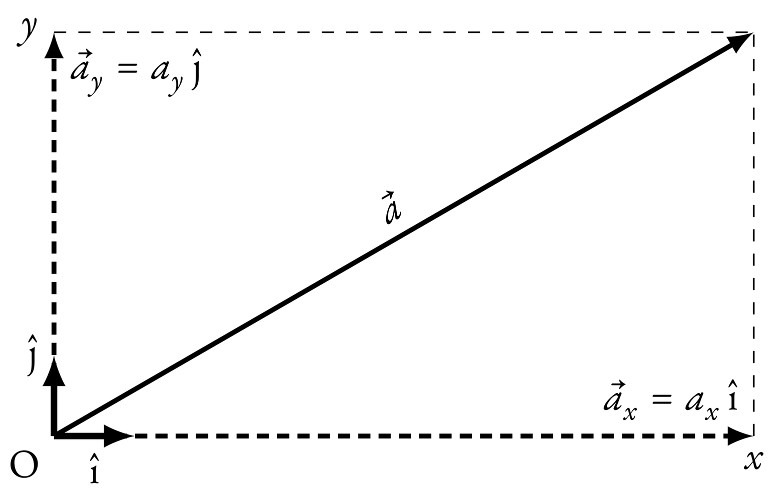
Forces as Vectors

Forces are **vector magnitudes**, which means that they are defined by a **vector**, which, in turn, is defined by:

Magnitude Also called modulus, it's the length of the line segment.

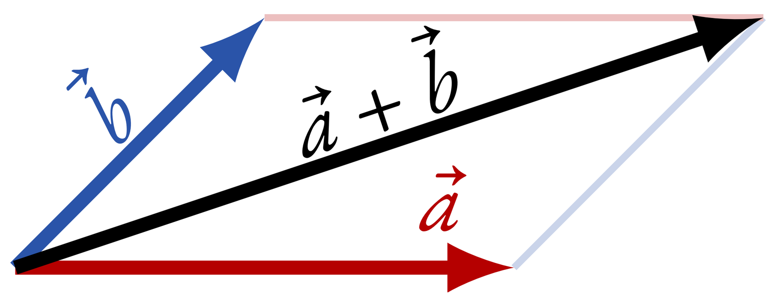
Direction The direction (angle) of the arrow from tail to head.



Two-dimensional vectors can be written as $\vec{a} = a_x\,\hat{\imath} + a_y\,\hat{\jmath}$, where $\hat{\imath}$ and $\hat{\jmath}$ are unit vectors (modulus = 1) on the x and y axis. The magnitude (modulus) of $\vec{a}$, $|\vec{a}|$, can be calculated using the Pythagorean Theorem $|\vec{a}| = \sqrt{a_x^2 + a_y^2}$.

Addition and subtraction of vectors

Graphically, we must draw one vector after the other and then join the initial and final points:



Mathematically, we add or subtract each coordinate (component) separately:

$$\vec{a} + \vec{b} = (a_x + b_x)\,\hat{\imath} + (a_y + b_y)\,\hat{\jmath}$$

Newton's Laws

1st Law (Law of Inertia)

"If no net force is applied to an object, the object will stay at rest or will move at a constant velocity."

2nd Law (Dynamic's Fundamental Law)

"The change in the state of motion of a body is directly proportional to, and in the same direction, as the net force that is acting upon that body."

Mathematically, it is written as

$$\sum \vec{F} = m\vec{a} \quad \text{(acceleration is directly proportional to the net force)}$$

In the **SI** force is measured in **newton** (N): $1\,\mathrm{N} = 1\,\mathrm{kg\,m\,s^{-2}}$.

3rd Law (Action-Reaction Principle)

"For every action there is always a reaction, of the same magnitude but in the opposite direction."

If a body A exerts a force over another body B, B will exert the same force on A but in the opposite direction ($\vec{F}_{\mathrm{AB}} = -\vec{F}_{\mathrm{BA}}$).

Forces of Special Interest

Weight $\vec{F}_\mathrm{w}$

Weight is the force exerted by the Earth on every object. It is calculated as:

$$\vec{F}_\mathrm{w} = m\vec{g},$$

where m is the mass of the object and $\vec{g}$ is the acceleration of gravity. It always points to the center of the Earth (downwards).

Normal Force $\vec{F}_\mathrm{n}$

Also known as **reaction** force, it is defined as the force exerted by a surface on every object standing on it. It has the same magnitude but opposite direction as the force exerted by the object on the surface.

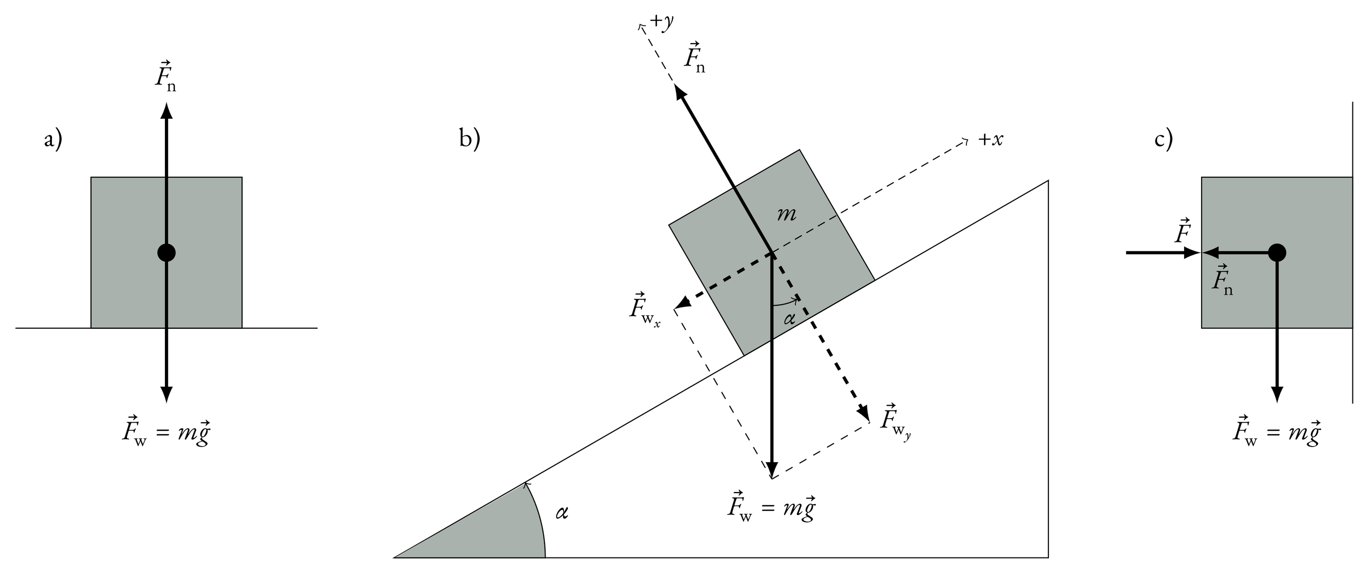


Figure 1. Normal force on a) a horizontal surface, b) a slide and c) a vertical surface.

Friction $\vec{F}_\mathrm{f}$

Friction force is the force between to surfaces in contact, opposing to the relative motion between both surfaces. The friction force is directly proportional to the normal force F_n:

$$F_\mathrm{f} = \mu F_\mathrm{n},$$

where μ es the coefficient of friction.

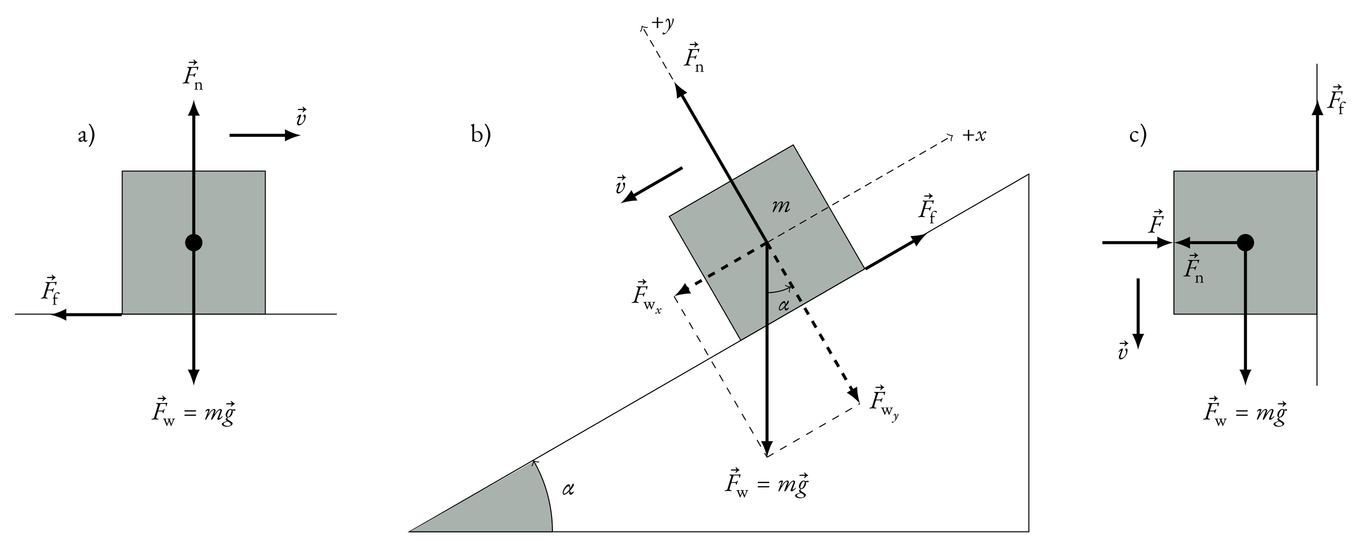


Figure 2. Friction force on a) a horizontal surface, b) a slide and c) a vertical surface.

Centripetal Force $\vec{F}_\mathrm{c}$

Centripetal force is the force or component of the force acting on an object moving on a curved path and pointing to the center of the trajectory. Its modulus is calculated using the **centripetal acceleration** and the **2nd Law of Newton**:

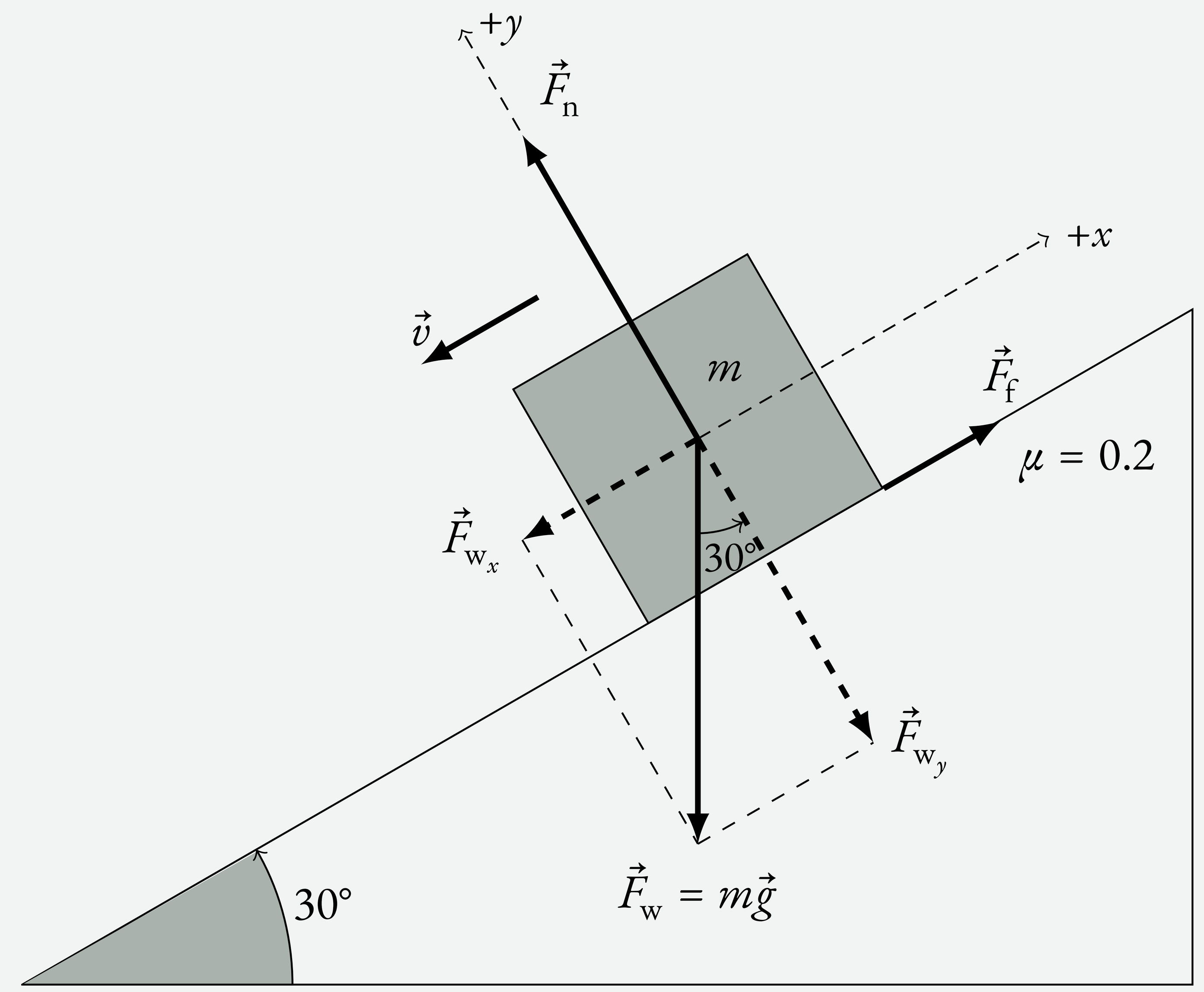
$$F_\mathrm{c} = ma_\mathrm{c} = m \cdot \frac{v^2}{R} = \frac{mv^2}{R}$$

Example

A body is falling down a slide inclined 30° with a coefficient of friction $\mu = 0.2$. Calculate the velocity and the space covered by the object after 5 s, if it was initially at rest.

Solution

First we make a drawing of the problem:



The **forces** acting are:

- Weight $\vec{F}_\mathrm{w} = -F_{\mathrm{w}_x}\,\hat{\imath} - F_{\mathrm{w}_y}\,\hat{\jmath}$, where:

$$F_{\mathrm{w}_x} = mg\sin\alpha = 9.8m\sin 30° = 4.9m\,\mathrm{N}$$
$$F_{\mathrm{w}_y} = mg\cos\alpha = 9.8m\cos 30° = 4.9\sqrt{3}m\,\mathrm{N}$$

- Normal $\vec{F}_\mathrm{n} = F_\mathrm{n}\,\hat{\jmath}$
- Friction force $\vec{F}_\mathrm{f} = \mu F_\mathrm{n}\,\hat{\imath} = 0.2F_\mathrm{n}\,\hat{\imath}\,\mathrm{N}$

We write the **2nd Law of Newton** for each **component**:

$$\textbf{Component } x \rightarrow F_\mathrm{f} - F_{\mathrm{w}_x} = ma \quad (1)$$
$$\textbf{Component } y \rightarrow F_\mathrm{n} - F_{\mathrm{w}_y} = 0 \quad (2)$$

Solving $F_\mathrm{n} = F_{\mathrm{w}_y} = 4.9\sqrt{3}m$ from (2) and substituting in (1), using also that $F_\mathrm{f} = 0.2N$ and that $F_{\mathrm{w}_x} = 4.9m$:

$$0.2 \cdot 4.9\sqrt{3}m - 4.9m = ma \rightarrow a = -3.2\,\mathrm{m/s^2}$$
$$\vec{a} = -3.2\,\hat{\imath}\,\mathrm{m/s^2}$$

The **velocity** after 5 s can be calculated with the **velocity equation**:

$$v = v_0 + at = 0 - 3.2 \cdot 5 = -16.0\,\mathrm{m/s}$$
$$\vec{v} = -16.0\,\hat{\imath}\,\mathrm{m/s}$$

For the **space covered** we can use the **motion equation**:

$$\Delta x = |x - x_0| = \left|v_0 \cdot t + \frac{1}{2}at^2\right| = \left|0 - \frac{1}{2} \cdot 3.2 \cdot 5^2\right| = 40.0\,\mathrm{m}$$